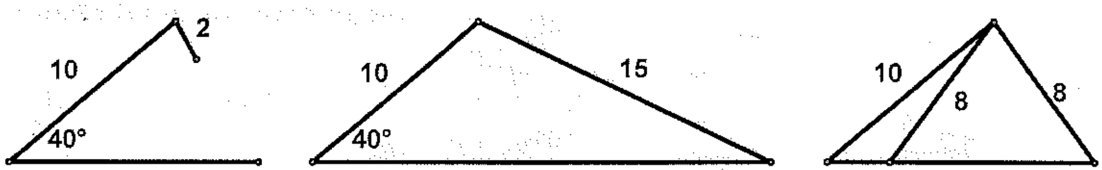
The SSA case (the ambiguous case)

Why is this ambiguous?
In Geometry, you learned that you could prove that two triangles were congruent using the following methods:

SSS
ASA
SAS
AAS

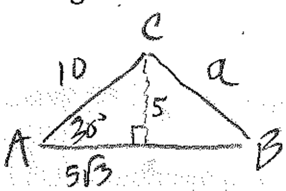
However, when you were given two sides and the NON-included angle (SSA) then, depending on the information given, you could construct 0, 1, or 2 triangles!
Here is what they might look like:



0 Triangles **1 Triangle** **2 Triangles**

So this means that **not just one** unique triangle can necessarily be created.

Now, how do we figure out if there are 0, 1, or 2 triangles with a SSA problem? Start by drawing a triangle with $m\angle A = 30°$ and $b = 10$.

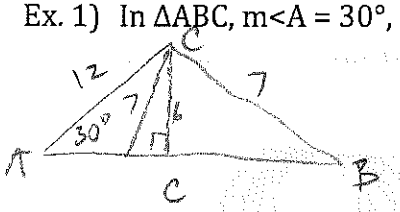


What do we know about side "a"? It is across from 30° ∠.
if a is altitude then $\sin 30 = \frac{a}{10}$
$\Rightarrow a = 5$

If $a < 5 \rightarrow$ 0 triangles
Now: *If* $a > 10 \rightarrow$ 1 triangle
If $5 < a < 10 \rightarrow$ 2 triangles

Section 6.1 – Law of SINES – Ambiguous Case

Ex. 1) In ΔABC, m<A = 30°, a = 7, and b = 12. Solve the triangle for all the missing sides and angles.



$\frac{\sin B}{12} = \frac{\sin 30}{7}$

$\sin B = 12\left(\frac{\sin 30}{7}\right) = \frac{12}{14} = \frac{6}{7} \approx .8571$

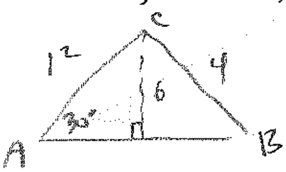
$\Rightarrow \boxed{B \approx 59°}$ or $180 - 59 = \boxed{121°}$

$\Rightarrow \boxed{C \approx 91°}$ or $\boxed{29°}$

If $C = 91°$, then $\frac{c}{\sin 91} = \frac{7}{\sin 30}$ $\Rightarrow \boxed{c \approx 14}$

$c = 29°$, then $\frac{c}{\sin 29} = \frac{7}{\sin 30}$ $\Rightarrow c \approx \boxed{6.79}$

Ex. 2) In ΔABC, m<A = 30°, a = 4, and b = 12. Solve the triangle for all the missing sides and angles.

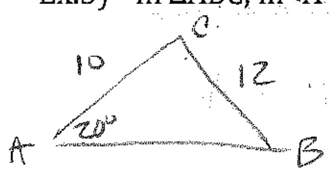


No solution

$\frac{\sin B}{12} = \frac{\sin 30}{4}$

$\sin B = \frac{12}{8} = \frac{3}{2}$ $\quad \sin B \le 1$!!

Ex.3) In ΔABC, m<A = 20°, a = 12, and b = 10. Solve the triangle for all the missing sides and angles.



1 Solution!

$\frac{\sin 20}{12} = \frac{\sin B}{10}$

$\sin B = 10\left(\frac{\sin 20}{12}\right) \approx .2850$

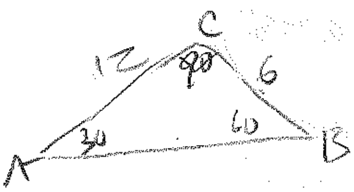
$\Rightarrow \boxed{B \approx 16.56°}$

$\Rightarrow \boxed{C \approx 143.44°}$

$\frac{c}{\sin 143.44} = \frac{12}{\sin 20}$

$\Rightarrow c = \sin 143.44\left(\frac{12}{\sin 20}\right) \approx \boxed{20.90}$

Ex.4) In ΔABC, m<A = 30°, a = 6, and b = 12. Solve the triangle for all the missing sides and angles.



1 solution!

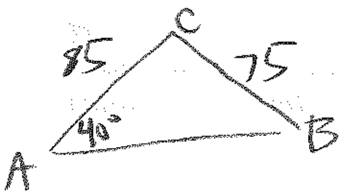
$\frac{\sin 30}{6} = \frac{\sin B}{12}$

$\boxed{B = 90°} \rightarrow \boxed{C = 60°}$

$c = 6\sqrt{3} \approx 10.4$

Section 6.1 – Law of SINES – Ambiguous Case

Ex.5) In ΔABC, m<A = 40°, a = 75, and b = 85. Solve the triangle for all the missing sides and angles.



$\frac{\sin B}{85} = \frac{\sin 40}{75}$

$\sin B = 85\left(\frac{\sin 40}{75}\right) \approx .7285$

$\Rightarrow \boxed{B \approx 46.76°} \Rightarrow C \approx 93.24°$

$\frac{75}{\sin 40} = \frac{c}{\sin 93.24}$

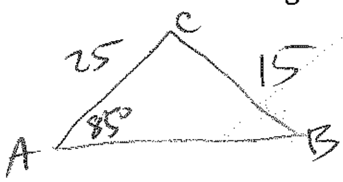
$\boxed{c \approx 116.49}$

or $\boxed{B \approx 133.24°} \Rightarrow C \approx 6.76°$

$\frac{75}{\sin 40} = \frac{c}{\sin 6.76}$

$\boxed{\Rightarrow c \approx 13.73}$

Ex.6) In ΔABC, m<A = 85°, a = 15, and b = 25.
Solve the triangle for all the missing sides and angles:

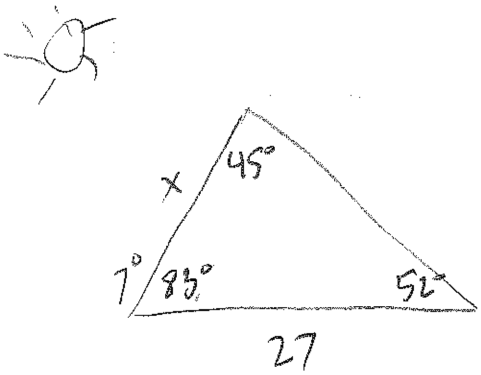


$\sin B = 25\left(\frac{\sin 85}{15}\right) \approx 1.66$

No answer!

Law of Sines: Applications!

1) A telephone pole tilts AWAY from the sun at a 7° angle from the vertical, and it casts a 27-foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is 52°. How tall is the pole?

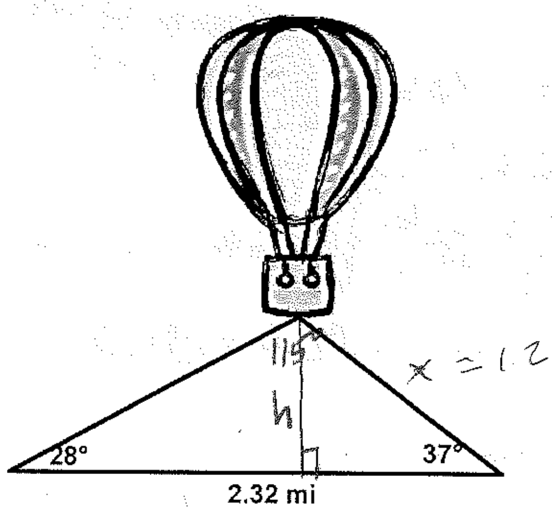


$\frac{x}{\sin 52°} = \frac{27}{\sin 45°}$

$x = \sin 52\left(\frac{27}{\sin 45}\right) \approx \boxed{30.09 \text{ ft}}$

Section 6.1 – Law of SINES – Ambiguous Case

2) Observers 2.32 miles apart see a hot-air balloon directly between them but at the angles of elevation shown in the figure. Find the altitude of the balloon:



$$\frac{x}{\sin 28} = \frac{2.32}{\sin 115^\circ}$$

$$x = \sin 28\left(\frac{2.32}{\sin 115}\right) \approx 1.20 \text{ m}$$

$$A = \tfrac{1}{2}(1.20)2.32 \sin 37^\circ \approx .8390$$

$$A = \tfrac{1}{2}bh = \tfrac{1}{2}(2.32)h = .8390$$

$$h = \frac{.8390 \times 2}{2.32} \approx \boxed{.7222 \text{ mi}}$$

$$\sin 37^\circ = \frac{h}{1.2}$$

$$h = 1.2 \sin 37^\circ \boxed{\approx .7222 \text{ mi}}$$