Section 6.1 – Law of SINES

So far, all the triangles we've solved have had one thing in common- they have all been ***right*** triangles. However, we can use sine and cosine to solve ***oblique*** triangles too (oblique = triangle WITHOUT a right angle).

To solve an oblique triangle, you must know the measure of at least one SIDE, and any two other parts of the triangle. So the possibilities are:

1) AAS
2) ASA
3) SSA
4) SSS
5) SAS

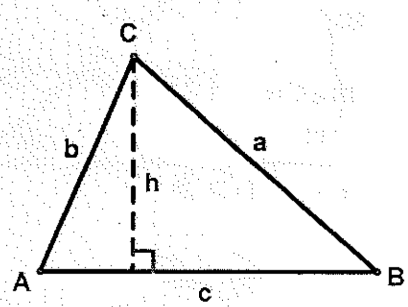
The first three of these situations can be solved with **Law of Sines** – the other two will use **Law of Cosines**. Today, we're going to discuss two of the first three.

Law of Sines

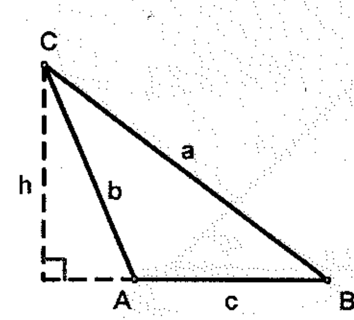
If $\triangle ABC$ has sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The triangle will look like one of the two shown below:



A is acute



A is obtuse

The Law of Sines can also be written in reciprocal form:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Section 6.1 – Law of SINES

Proof

Let's see why the Law of Sines is true. Considering the triangles show above, you can see that

$$\sin A = \frac{h}{b} \text{ or } h = b\sin A \text{, and}$$

$$\sin B = \frac{h}{a} \text{ or } h = a\sin B$$

From this, $b\sin A = a\sin B$

or

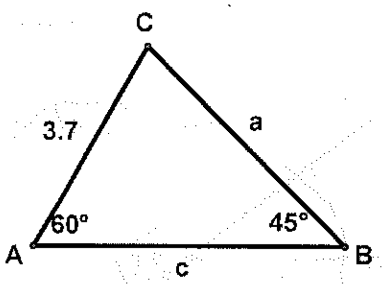
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

In a similar manner (you'd need an altitude from B to side $\overline{AC}$), you should be able to show that $\frac{c}{\sin C}$ equals the other two as well.

The AAS Case:

For the triangles below, find the remaining sides and angles

1)



m∠C= $180-60-45=75°$

a= 4.53

c= 5.05

$$\frac{a}{\sin 60} = \frac{3.7}{\sin 45}$$

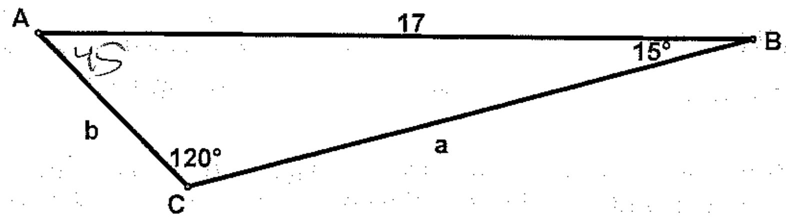
$$a = \frac{3.7\sin 60}{\sin 45} \approx 4.53$$

$$\frac{c}{\sin 75} = \frac{3.7}{\sin 45}$$

$$c = \frac{3.7\sin 75}{\sin 45} \approx 5.05$$

Section 6.1 – Law of SINES

2)



$m\angle A = 45°$

$a = 13.88$

$b = 5.08$

$$\frac{17}{\sin 120} = \frac{b}{\sin 15}$$

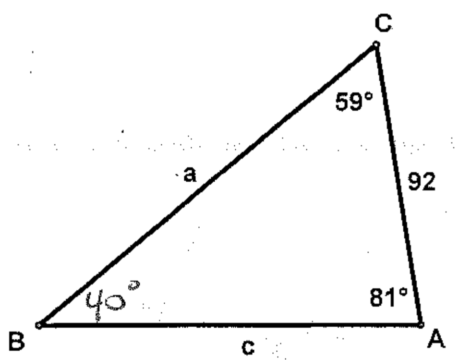
$$b = \frac{17 \sin 15}{\sin 120} \approx 5.08$$

$$\frac{17}{\sin 120} = \frac{a}{\sin 45}$$

$$a = \frac{17 \sin 45}{\sin 120} \approx 13.88$$

The ASA case:

3)



$m\angle B = 40°$

$a = 141.36$

$c = 122.68$

$$\frac{a}{\sin 81} = \frac{92}{\sin 40}$$

$$a = \frac{92 \sin 81}{\sin 40} \approx 141.36$$

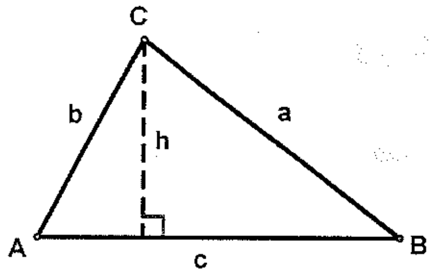
$$\frac{c}{\sin 59} = \frac{92}{\sin 40}$$

$$c = \frac{92 \sin 59}{\sin 40}$$

$$\approx 122.68$$

Section 6.1 – Law of SINES

Area of an oblique triangle



The area of $\triangle ABC =$ $\frac{1}{2}bh$

Now, play around and see if you can get h in terms of the sides a, b, and c.

In terms of a, h = $a \sin B$

In terms of b, h = $b \sin A$

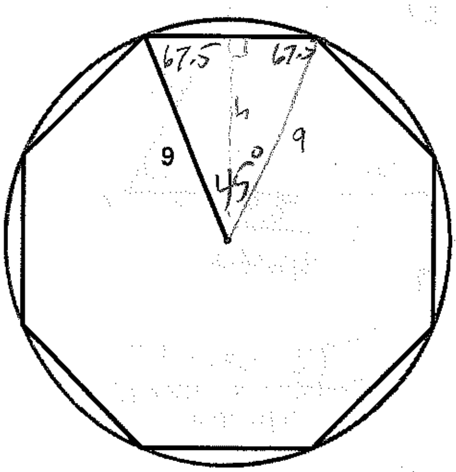
Therefore, using c as the base, the area of $\triangle ABC =$ $\frac{1}{2} ca \sin B$

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of the two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$$

Example: Find the AREA of a regular octagon (equiangular and equilateral) inscribed in a circle of radius 9 inches:



$\frac{360}{8} = 45$

$A = \left(\frac{1}{2} 9 \cdot 9 \sin 45\right) 8$

$= \left(\frac{81}{2} \cdot \frac{\sqrt{2}}{2}\right) 8^{4} = \frac{324\sqrt{2}}{2} = 162\sqrt{2}$

≈ 229.11