Section 6.1 – Law of SINES

So far, all the triangles we've solved have had one thing in common- they have all been ***right*** triangles. However, we can use sine and cosine to solve ***oblique*** triangles too (oblique = triangle WITHOUT a right angle).

To solve an oblique triangle, you must know the measure of at least one SIDE, and any two other parts of the triangle. So the possibilities are:

1) AAS
2) ASA
3) SSA
4) SSS
5) SAS

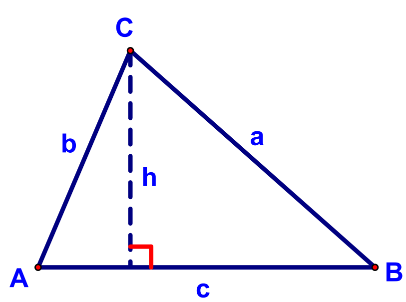
The first three of these situations can be solved with **Law of Sines** – the other two will use **Law of Cosines**. Today, we're going to discuss two of the first three.

Law of Sines

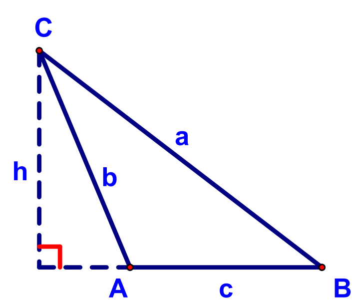
If $\triangle ABC$ has sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The triangle will look like one of the two shown below:



A is acute



A is obtuse

The Law of Sines can also be written in reciprocal form:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

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Proof

Let's see why the Law of Sines is true. Considering the triangles show above, you can see that

$$\sin A = \frac{h}{b} \text{ or } h = b\sin A \text{, and}$$

$$\sin B = \frac{h}{a} \text{ or } h = a\sin B$$

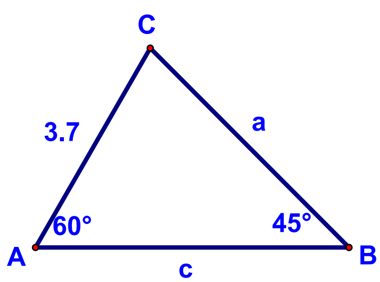
From this,

In a similar manner (you'd need an altitude from B to side $\overline{AC}$), you should be able to show that $\frac{c}{\sin C}$ equals the other two as well.

The AAS Case:

For the triangles below, find the remaining sides and angles

1)



$m\angle C =$

$a =$

$c =$

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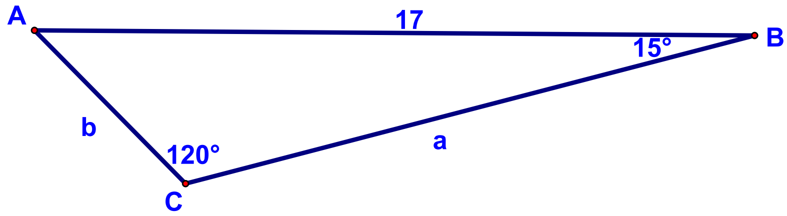
2)



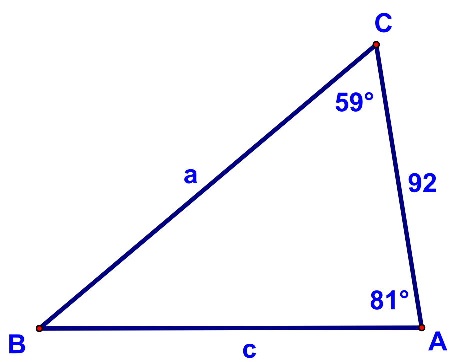
m∠A =

a =

b =

The ASA case:

3)



m∠B =

a =

c =

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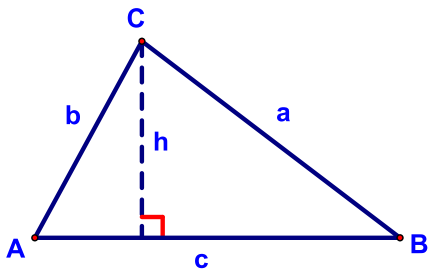
Area of an oblique triangle



The area of $\triangle ABC =$

Now, play around and see if you can get h in terms of the sides a, b, and c.

In terms of a, h =

In terms of b, h =

Therefore, using c as the base, the area of $\triangle ABC =$

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of the two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

Example: Find the AREA of a regular octagon (equiangular and equilateral) inscribed in a circle of radius 9 inches: