Section 6.2 – Law of COSINES

The two remaining cases for solving oblique triangles, SSS and SAS, cannot be solved using the Law of Sines because none of the ratios would be complete. In these two cases, the Law of Cosines must be used.

Law of Cosines

Standard Form – use with SAS

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Alternative Form – use with SSS

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

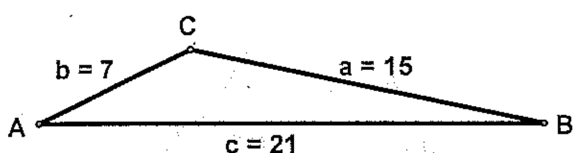
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

** TRY TO FIND THE LARGEST ANGLE FIRST **

SSS Case

1) Find the three angles of the triangle in the figure below



$m\angle A =$ 25.66°

$m\angle B =$ 11.66°

$m\angle C =$ 142.68°

$$\cos A = \frac{7^2 + 21^2 - 15^2}{2(7 \cdot 21)} = \frac{265}{294}$$

$$\Rightarrow A \approx 25.66$$

$$\cos B = \frac{15^2 + 21^2 - 7^2}{2(15)(21)} = \frac{617}{630}$$

$$\Rightarrow B \approx 11.66$$

$$\cos C = \frac{7^2 + 15^2 - 21^2}{2(7 \cdot 15)} = -\frac{167}{210}$$

$$C \approx 142.68$$

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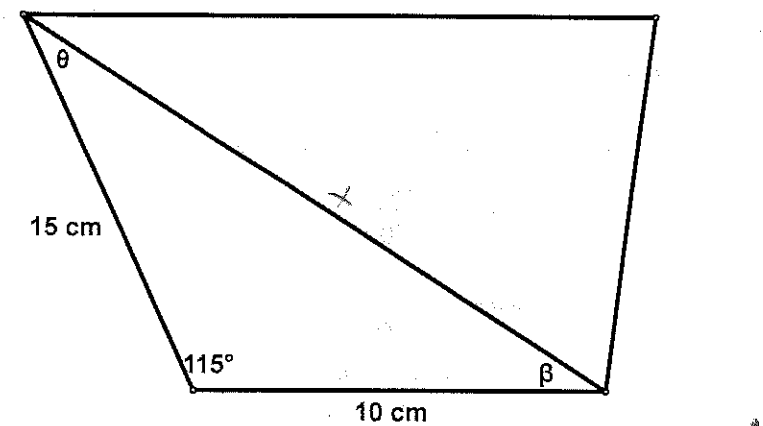
SAS Case

2) Find the length of the diagonal of the trapezoid, and the missing angles.



θ= 25.25°

β= 39.75°

diagonal= 21.26

$x^2 = 15^2 + 10^2 - 2(15)(10)\cos 115°$

≈ 451.79

$\Rightarrow x \approx 21.26$

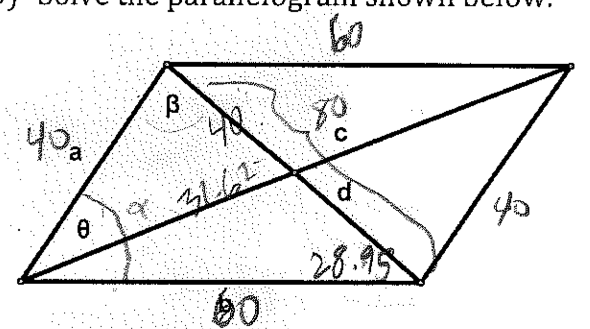
$\frac{15}{\sin\beta} = \frac{21.26}{\sin 115}$

$\sin\beta = 15\left(\frac{\sin 115}{21.26}\right)$

$\approx .63945$

$\Rightarrow \beta \approx 39.75$

3) Solve the parallelogram shown below.



* c and d are diagonals of the parallelogram

a=40

b=60

d=80

c= 63.24

θ= 66.72

β= 46.57°

$60^2 = 40^2 + 80^2 - 2(40\cdot 80)\cos\beta$

$\cos\beta = \frac{60^2 - (40^2 + 80^2)}{-2(40\cdot 80)} = \frac{11}{16}$

$\Rightarrow \beta \approx 46.57°$

$80^2 = 40^2 + 60^2 - 2(40\cdot 60)\cos\alpha$

$\cos\alpha = \frac{80^2 - (40^2 + 60^2)}{-2\cdot 40\cdot 60} = -\frac{1}{4}$

$\Rightarrow \alpha \approx 104.48$

$c^2 = 40^2 + 60^2 - 2(40\cdot 60)\cos 75.52$

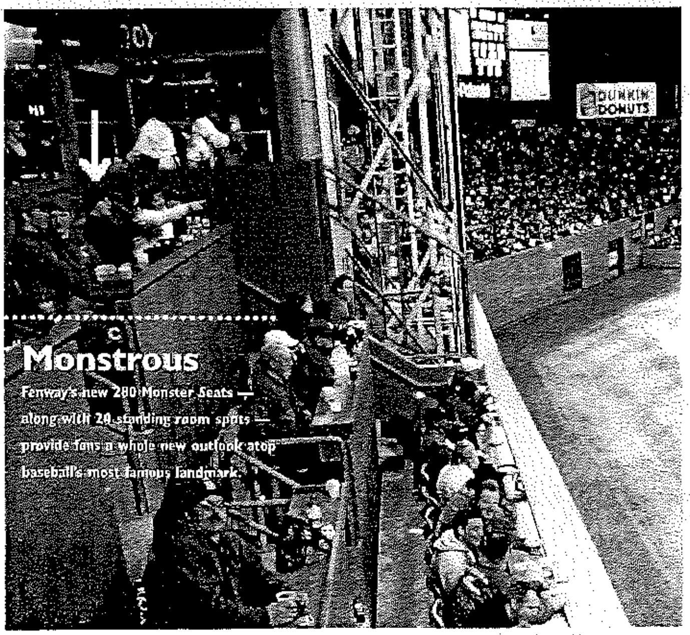
≈ 3999.80

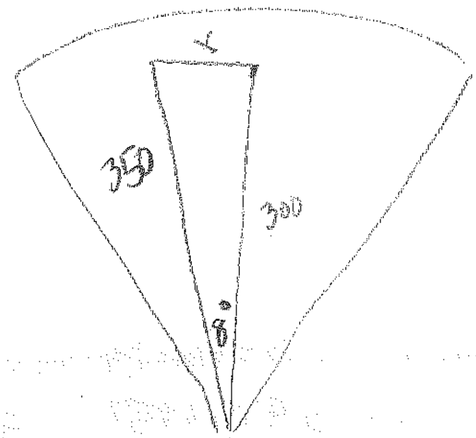
$c \approx 63.24$

$\theta = (180 - 46.57)/2 \approx 66.72$ } because Δ is 1so's

Section 6.2 – Law of COSINES

4) Jacoby Ellsbury is playing centerfield against the Yankees. He is approximately 300 ft. from the television camera that is behind home plate. Derek Jeter hits a fly ball that goes to the green monster 350 ft. away from the camera. The camera turns 8° to follow the play. Approximately how far does Ellsbury have to run to the make the catch?





$x^2 = 300^2 + 350^2 - 2(300)(350)\cos 8°$

$x^2 \approx 4543.71$

$\Rightarrow x \approx 67.41$ ft

Section 6.2 – Law of COSINES

Heron's Area Formula

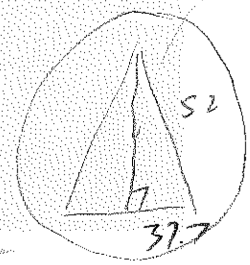
Heron's Area Formula

Given any triangle with sides of lengths a, b, and c, the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

5) Find the area of a triangle having sides of length a = 75.4, b = 52, and c = 52.



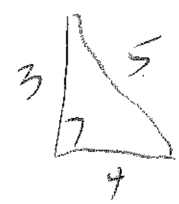
$$S = \frac{75.4+52+52}{2} = 89.7$$

$$A = \sqrt{89.7(89.7-75.4)(89.7-52)(89.7-52)}$$

$$= \sqrt{89.7(14.3)(37.7)^2} \approx \boxed{1350.22\ u^2}$$

6) Find the area of a triangle with sides of length a = 3, b = 4, and c = 5

$$S = \frac{3+4+5}{2} = 6 \qquad A = \sqrt{6(1)(2)(3)} = \sqrt{36} = \boxed{6u^2}$$



$A = \frac{1}{2}4\cdot 3 = 6$

Is there a way to check your answer for this one?

$$A = \tfrac{1}{2}bh = \tfrac{1}{2}3\cdot 4 = 6u^2$$

WHICH area formula to use?? (Recall: we learned another one with Law of Sines)

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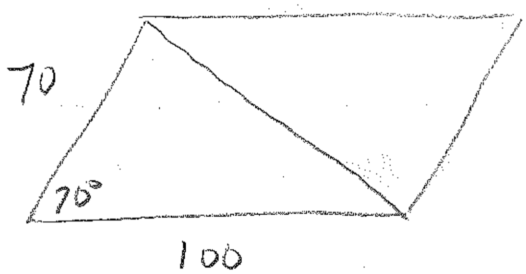
7) A parking lot has the shape of a parallelogram. The lengths of the two adjacent sides are 70 meters and 100 meters. The angle between the two sides is 70°. What is the area of the parking lot?



$$A = 2\left(\tfrac{1}{2}(70)(100)\sin 70\right)$$

$$= 7000 \sin 70^\circ \approx \boxed{6577.85 \text{ m}^2}$$