

Warm Up: Solve the system of equations.

a)  $x - y = -4$     $x = y - 4$   
 $x + 2y = 5$   
 $y - 4 + 2y = 5$   
 $y = 3$   
 $x = 3 - 4 = -1$

b)  $6x - 3y - 4 = 0$   
 $x + 2y - 4 = 0$   
 $x = -2y + 4$   
 $6(-2y + 4) - 3y = -4$   
 $-12y + 24 - 3y = -4$   
 $20 = 15y$   
 $\frac{4}{3} = y$   
 $x = -2\left(\frac{4}{3}\right) + 4$   
 $= -\frac{8}{3} + \frac{12}{3} = \frac{4}{3}$

Examples

Two Solution Case

a)  $x - y = -4 \rightarrow x = y - 4$   
 $x^2 - y = -2$   
 $(y - 4)^2 - y = -2$   
 $y^2 - 8y + 16 - y = -2$   
 $y^2 - 9y + 18 = 0$   
 $(y - 6)(y - 3) = 0$   
 $y = 6, 3$   
 $x = 2, -1$   
 $(2, 6) \text{ \& } (-1, 3)$

b)  $x^2 + y = 0$     $y = -x^2$   
 $x^2 - 4x - y = 0$   
 $x^2 - 4x + x^2 = 0$   
 $2x^2 - 4x = 0$   
 $2x(x - 2) = 0$   
 $x = 0, 2$   
 $y = 0, -4$   
 $(0, 0) \text{ \& } (2, -4)$

## No Real Solution Case

$$a) \quad -\frac{2}{3}x + y = 2 \rightarrow y = \frac{2x}{3} + 2$$

$$2x - 3y = 6$$

$$2x - 3\left(\frac{2x}{3} + 2\right) = 6$$

$$2x - 2x - 6 = 6$$

$$-6 = 6$$

NO SOLN!

$$b) \quad x + y = 4$$

$$y = -x + 4$$

$$x^2 + y = 2$$

$$x^2 + (-x + 4) = 2$$

$$x^2 - x + 2 = 0$$

not factorable, so

$$-(-1) \pm \sqrt{1^2 - 4(1)(2)}$$

$$= \frac{1 \pm \sqrt{1 - 8}}{2} = \text{no real soln!}$$

## Applications

A total of \$20,000 is invested in two funds paying 6.5% and 8.5% simple interest. The investor wants a yearly interest check of \$1600 from the two investments. Write and solve a system of equations to determine how much is invested at each interest rate.

$$x + y = 20,000 \quad x = 20,000 - y$$

$$.065x + .085y = 1600$$

$$.065(20,000 - y) + .085y = 1600$$

$$1300 - .065y + .085y = 1600$$

$$.02y = 300$$

$y = 15,000$	$\rightarrow 8.5\%$
$x = 5,000$	$\rightarrow 6.5\%$

A small fast-food restaurant invests \$5,000 to produce a new food item that will sell for \$3.49. Each item can be produced for \$2.16.

- a) How many units must be sold to break even?  
 b) How many units must be sold to make a profit of \$8500?

$$R = 3.49x \quad (\text{Revenue})$$

$$C = 5,000 + 2.16x \quad (\text{Cost})$$

a)

$$3.49x = 5,000 + 2.16x$$

$$1.33x = 5,000$$

$$x \approx 3760 \text{ units}$$

b)

$$P = R - C = 3.49x - (5,000 + 2.16x)$$

$$8500 = 1.33x - 5,000$$

$$13,500 = 1.33x$$

$$x \approx 10,151 \text{ units}$$

The perimeter of a rectangle is 280 cm and the width is 20 cm less than the length. Find the dimensions of the rectangle.



$$2l + 2(l-20) = 280$$

$$2l + 2l - 40 = 280$$

$$4l = 320$$

$$l = 80$$

$$w = 80 - 20 = 60$$