Pre-Calculus CP 1 – Section 8.2 Notes Name: KEY

Matrix Multiplication

In order to MULTIPLY two matrices, the number of COLUMNS in the **first** matrix must match the number of ROWS in the **second** matrix.

For example: If A is an m x n matrix and B is an n x p matrix,
then you **can** multiply because the "n"s are equal, and the product is a m x p matrix

Matrix Multiplication

A	x	B	=	AB
m x n	x	n x p	=	m x p

Ex. 1) State whether the product is defined (**can** you multiply?)
If so, give the dimensions of AB

a) A: 2 x 3 B: 3 x 4 AB: 2X4

b) A: 3 x 2 B: 3 x 4 AB: DNE.

c) A: 5 x 4 B: 5 x 4 AB: DNE

d) A: 4 x 4 B: 4 x 5 AB: 4X5

Ex. 2) Find the product of the matrices

$A: \begin{bmatrix} 5 & 4 & 2 \end{bmatrix}$ (1 x 3)

$B: \begin{bmatrix} 6 & 1 \\ 3 & -2 \\ -4 & 7 \end{bmatrix}$ (3 x 2)

1 x 2

$AB = \begin{bmatrix} 5\cdot6+4\cdot3+2\cdot-4 & 5\cdot1+4\cdot-2+2\cdot7 \end{bmatrix}$

$= \begin{bmatrix} 30+12-8 & 6-8+14 \end{bmatrix} = \begin{bmatrix} 34 & 11 \end{bmatrix}$

Ex. 3) Find each product

A: $\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ 2×2

B: $\begin{bmatrix} 1 & -4 \\ 2 & 1 \end{bmatrix}$ 2×2 $= 2 \times 2$

AB: $\begin{bmatrix} 3\cdot1+2\cdot2 & 3\cdot-4+2\cdot1 \\ -1\cdot1+0\cdot2 & -1\cdot-4+0\cdot1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -1 & 4 \end{bmatrix}$

BA: $\begin{bmatrix} 1\cdot3+-4\cdot-1 & 1\cdot2+-4\cdot0 \\ 2\cdot3+1\cdot-1 & 2\cdot2+1\cdot0 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 5 & 4 \end{bmatrix}$

Does AB = BA? NO

Is matrix multiplication commutative? (is order unimportant?) NO

Ex. 4) Find the product of the matrices

A: $\begin{bmatrix} -2 & 3 \\ 1 & -4 \\ 6 & 0 \end{bmatrix}$ 3×2

B: $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ 2×2 $= 3 \times 2$

$AB = \begin{bmatrix} -2\cdot-1+3\cdot-2 & -2\cdot3+3\cdot4 \\ 1\cdot-1+-4\cdot-2 & 1\cdot3+-4\cdot4 \\ 6\cdot-1+0\cdot-2 & 6\cdot3+0\cdot4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ 7 & -13 \\ -6 & 18 \end{bmatrix}$

Properties of Matrix Operations: A, B, and C are matrices, and k is a scalar

- Associative Property of Addition: A + (B + C) = (A+B) + C
- Commutative Property of Addition: A + B = B + A
  - Does the commutative property work for subtraction too? NO
  - Does the commutative property work for multiplication? NO
- Distributive Property of Addition (with a scalar): k(A + B) = kA + kB

  AND Subtraction: k(A – B) = kA – kB
- Associative Property of Multiplication: A(BC) = (AB)C
- Distributive Property of Multiplication: A(B + C) = AB + AC

  Or (A + B)C = AC + BC
- Associative Property of scalar multiplication: k(AB) = (kA)B or A(kB)

Ex. 5) Using matrix operations, find and simplify the following:

A: $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ B: $\begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix}$ C: $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

a) A(B + C)

$= \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 7 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2\cdot -1 + 1\cdot 7 & 2\cdot 1 + 1\cdot 4 \\ -1\cdot -1 + 3\cdot 7 & -1\cdot 1 + 3\cdot 4 \end{bmatrix}$

$= \begin{bmatrix} 5 & 6 \\ 22 & 11 \end{bmatrix}$

b) B – C + A

$\begin{bmatrix} -3 & -1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

Word Problem!! Two softball teams submit equipment lists for the season which are shown below.

Women's Team	Men's Team
12 bats	15 bats
45 balls	38 balls
15 uniforms	17 uniforms

Each bat costs $21, each ball costs $4, and each uniform costs $30.

Write two matrices, one to represent the equipment requested and one to represent cost. LABEL your rows and columns- this is very important!

$$\begin{matrix} & \text{Bat} & \text{Ball} & \text{U} \\ \text{W} \\ \text{M} \end{matrix}\begin{bmatrix} 12 & 45 & 15 \\ 15 & 38 & 17 \end{bmatrix} \qquad \begin{matrix} & \$ \\ \text{Bat} \\ \text{Ball} \\ \text{uni} \end{matrix}\begin{bmatrix} 21 \\ 4 \\ 30 \end{bmatrix}$$

2×3 $\qquad$ 3×1

2×1

Find the total cost of equipment for each team:

$$\begin{matrix} & \$ \\ \text{W} \\ \text{M} \end{matrix}\begin{bmatrix} 12 \cdot 21 + 45 \cdot 4 + 15 \cdot 30 \\ 15 \cdot 21 + 38 \cdot 4 + 17 \cdot 30 \end{bmatrix}$$

$$\begin{matrix} & \$ \\ \text{W} \\ \text{M} \end{matrix}\begin{bmatrix} 882 \\ 977 \end{bmatrix}$$

HW: p 597-601, #27, 29, 34, 37, 40, 43, 45, 49