**Pre-Calculus CP 1 – Section 8.3/8.4 Notes** **Name:** KEY

**Determinants, Inverses, and the Identity Matrix**

Determinants are used to help you find the INVERSE of a matrix, and the inverse of a matrix will help you solve a system of equations!

The notation for a determinant looks like the absolute value notation:

$$\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} \text{ means find the determinant for matrix } \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

Here's the formula for a 2x2:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Let's try a few determinants before we see how the determinants are used.

Ex.1) $\begin{vmatrix} 7 & 2 \\ 2 & 3 \end{vmatrix} = 21 - 4 = 17$

Ex. 2) $\begin{vmatrix} -5 & 1 \\ -7 & 4 \end{vmatrix} = -20 - -7 = -13$

A 3x3 is a bit more complicated. Let's do these by hand so you will appreciate the calculator magic that much more:

Ex. 3) $\begin{vmatrix} 4 & 3 & 1 \\ 5 & -7 & 0 \\ 1 & -2 & 2 \end{vmatrix} = 4\begin{vmatrix} -7 & 0 \\ -2 & 2 \end{vmatrix} - 3\begin{vmatrix} 5 & 0 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 5 & -7 \\ 1 & -2 \end{vmatrix}$

$= 4(-14-0) - 3(10-0) + -10 - -7$

$= -56 - 30 - 3 = -89$

Ex.4) $\begin{vmatrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ 1 & 2 & 4 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 2 & 4 \end{vmatrix} - -1\begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} -2 & 0 \\ 1 & 2 \end{vmatrix}$

$= 2(0-2) + (-8-1) + 3(-4-0)$

$= -4 - 9 - 12 = -25$

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## Determinants on the Calculator!!

***Keystrokes:***

1. 2nd matrix
2. arrow to the right for MATH
3. #1 det(
4. 2nd matrix
5. enter the matrix letter that you want to find the determinant of (probably A)

Use your calculator to find the following determinants:

Ex.1) $\begin{vmatrix} -5 & 1 \\ -7 & 4 \end{vmatrix} = -13$

Ex. 2) $\begin{vmatrix} 4 & 3 & 1 \\ 5 & -7 & 0 \\ 1 & -2 & 2 \end{vmatrix} = -89$

## Using the determinant:

Use #1: Find the area of a triangle: the formula is $\text{Area} = \pm\frac{1}{2} \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

where the coordinates of the triangle go in the positions with x and y.

Example: Find the area of a triangle using the determinant formula and the coordinates (1,2), (6,2) and (4,0):

$$\pm\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 6 & 2 & 1 \\ 4 & 0 & 1 \end{vmatrix} = \frac{10}{2} = 5u^2$$

***You try:*** Find the area of a triangle using the determinant formula and the coordinates (3,9), (4,-2) and (0,5):

$$\pm\frac{1}{2}\begin{vmatrix} 3 & 9 & 1 \\ 4 & -2 & 1 \\ 0 & 5 & 1 \end{vmatrix} = -\frac{1}{2}(-37) = \frac{37}{2}u^2$$

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Use #2: Finding an INVERSE

The inverse of a 2 x 2 matrix, $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, can be found using the following fomula:

$$A^{-1}=\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples- find the inverse of the following by hand:

1) $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$inv = \frac{1}{6-4}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$

2) $\begin{bmatrix} -4 & 3 \\ -3 & 2 \end{bmatrix}$

$inv = \frac{1}{-8+9}\begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}$

To find the inverse on the graphing calculator, input your matrix into matrix A, then press:

1. 2nd matrix
2. enter for A
3. $x^{-1}$ button
4. enter

We will not do 3 x 3 inverse matrices by hand- instead we will do them on the calculator!
Try these in your calculator (hit math-frac if you get crazy decimals) :

3) $\begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & -1 \\ -1 & 3 & -3 \end{bmatrix}$

$inv = \begin{bmatrix} 3 & 0 & -1 \\ 7 & -1 & -2 \\ 6 & -1 & -2 \end{bmatrix}$

4) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & -2 \\ 3 & 1 & 1 \end{bmatrix}$

$inv = \begin{bmatrix} 1 & 1 & -1 \\ -9 & -8 & 11 \\ 6 & 5 & -7 \end{bmatrix}$

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## The IDENTITY matrix:

The identity matrix is like multiplying by 1 – if you multiply a matrix by the identity, it will stay unchanged. Identity matrices can only be square, or n x n. Two examples are:

2 x 2: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  3 x 3: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<u>If you ever want to check to ensure that you have the right inverse matrix,</u>

Two matrices are INVERSES of each other if their **product** is the identity matrix

Ex: $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 3\cdot2+-1\cdot5 & 3\cdot1+-1\cdot3 \\ -5\cdot2+2\cdot5 & -5\cdot1+2\cdot3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ex: Are the following two matrices inverses of each other? Find the product to prove your answer:

$\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1.5 & -3 \end{bmatrix} = \begin{bmatrix} 5\cdot-1+-3\cdot1.5 & 5\cdot2+-3\cdot-3 \\ -4\cdot-1+2\cdot1.5 & -4\cdot2+2\cdot-3 \end{bmatrix}$

NO! $= \begin{bmatrix} -9.5 & 19 \\ 7 & -14 \end{bmatrix}$ NOT inverses!

HW: p. 608 #1, 3, 5, 13, 17, 21, 31 **and** p. 616 # 5, 15, 21 **and** p. 628 #15, 29