Bell Work

Recalling our work in 9.5, answer the following questions.

1) Find the value of 5! (by hand) $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

2) Find the value of 10! on your calculator. (Math→ PROB→ #4)
   3,628,800

3) Find the value of ${}_5C_2$ on your calculator. Note that your textbook will display also as $\binom{5}{2}$ and also as $C(5,2)$. 10

4) In the previous question you found the numerical value of what scenario? In other words, what does ${}_5C_2$ represent? Select 2 from a group of 5

"5 choose 2"

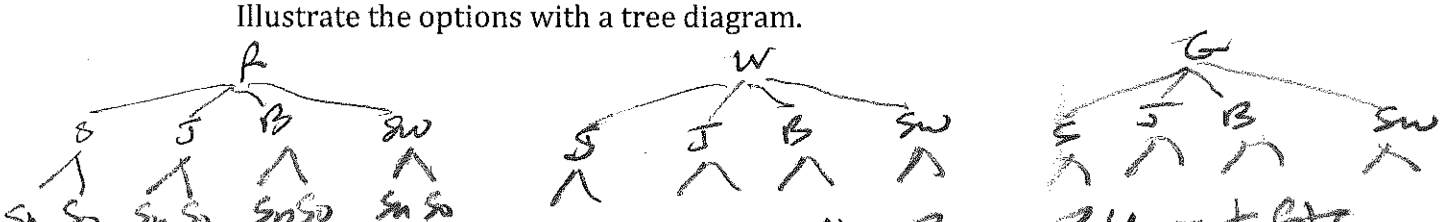
Collection of 5 things taken 2 at a time

Counting Principle

Used to determine in how many ways something can occur.

- The number of ways an event that consists of a bunch of choices for smaller events can occur is equal to the PRODUCT of the number of choices for each smaller event.
  - In other words, if there are *m* ways to perform one event and *n* ways for another event to occur then there are *m x n* ways for both events to occur. This can be applied to more than two events.

Example: Sophie has three shirts (red, white, green); four pants (shorts, jeans, black pants, sweat pants); and two pairs of shoes (sneakers, sandals) in her bag. How many outfits can she create? Assume she wears one shirt, one pant, and one pair of shoes. Illustrate the options with a tree diagram.



$3 \cdot 4 \cdot 2 = 24$ outfits

Example: You are designing your new car. You have ten choices for color, three choices for tires, and a choice of either a coupe (2-door) or a sedan (4-door). How many different cars can you design? Assume only one of each is selected!

$10 \cdot 3 \cdot 2 = 60$

Permutations & Combinations

- The number of ways that an event can occur is viewed as a
  - Permutation (P) if order DOES MATTER
  - Combination (C) if order DOES NOT MATTER.

- **How to determine if order matters:** Ask yourself if changing the order yields an outcome that should be considered different and thus counted again.

Permutations (order matters!)

Definition: An **ordered** (line) arrangement of "r" objects chosen from "n" objects.

- One approach that often works for permutations is to use **dashes** and the counting principle

Types of <u>linear</u> permutations:

Type #1: When you use the "P" on the calculator: (Distinct, with repetition)
The "n" objects are distinct,
and repetition is allowed in the selection of "r" of them

Example: Suppose you have a set of four objects: A, B, C, and D. List out all the different ways there are to select two from these four if order matters, i.e. AB and BA are different!

AB	BA	CA	DA
AC	BC	CB	DB
AD	BD	CD	DC

- There are 4 choices for the first selection and 3 choices for the second selection so there must be 4·3 possibilities.

- Find ${}_4P_2$ on the calculator and compare this to your previous answer.

  12 ... it's the same!

$P(n,r) = {}_nP_r = \dfrac{n!}{(n-r)!}$ is used to determine the number of ways of selecting *r* objects from *n* ***distinct*** objects when <u>repetition is not allowed</u> and order matters.

Type #2: When you use dashes: (Distinct, without repetition)
The "n" objects are distinct,
and repetition is NOT allowed in the selection of "r" of them

Example: Airport codes consist of three letters. For example, BOS stands for Boston and FLL stands for Fort Lauderdale.

a) How many possible airport codes exist?

$\underline{26} \cdot \underline{26} \cdot \underline{26} = 17,576$

b) How many possible airport codes would exist if they did NOT allow repetition?

$\underline{26} \cdot \underline{25} \cdot \underline{24} = 15,600$

${}_{26}P_3 = \frac{26!}{23!}$

Combinations (order DOES NOT matter!)

Example: Suppose that from A, B, C, and D, you need to pick two letters.
List out all the possible outcomes:

AB BC CD
AC BD
AD

${}_4C_2 = 6$

$C(n,r) = {}_nC_r = \dfrac{n!}{r!(n-r)!}$ is used to determine the number of ways selecting *r* objects from *n* distinct objects <u>without repetition</u> with no regard to order!

Conceptual Question: Why is ${}_4P_2$ larger than ${}_4C_2$?

order matters ... no "repeats"
∴ AB & BA are different

Lots of Examples!

#1: Suppose 80 people entered a raffle and three people are going to be selected and each will win a brand new Lexus. How many ways could the three winners be selected?

$_{80}C_3 = 82,160$

#2: Suppose 80 people entered a raffle and three people are going to be selected to win either a new Lexus, a new Honda Civic, or a new bicycle. How many ways could the three winners be selected?

$\underset{L}{\underline{80}} \cdot \underset{C}{\underline{79}} \cdot \underset{B}{\underline{78}} = {}_{80}P_3 = 492,960$

#3: A vase contains 10 pink tulips and 5 yellow tulips. Suppose you want to pick seven of them and have three that are pink. How many ways can you do this?

$_{10}C_3 \cdot {}_5C_4 = 120 \cdot 5 = 600$

#4: From a class of 20 students, a teacher will pick two students to give an award to. In how many ways can this selection be made?

$_{20}C_2 = 190$

#5: From a class of 20 students, a teacher will pick three and assign the first an A, the second a B, and the third a C for the term. In how many ways can this selection be made?

$\underset{A}{\underline{20}} \cdot \underset{B}{\underline{19}} \cdot \underset{C}{\underline{18}} = {}_{20}P_3 = 6,840$

#6: A security code consists of five digits that can not be repeated. How many possible codes exist?

$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} = {}_{10}P_5 = 30,240$

#7: You take a test with 20 True/False questions. In how many different ways can the answer sheet be filled out?

$2^{20} = 1,048,576$

#8: How many ways can you form a committee of 2 faculty members and 3 students to help form the DS Mission Statement? There are 6 faculty members and 10 students who have volunteered to serve on such a committee.

$_6C_2 \cdot {}_{10}C_3 = 1,800$

Back to Permutations

Type #3: (Not distinct, use all in arrangement)-- **(CANNOT use nPr)**

The "n" objects are NOT distinct,
and we use all of them in the arrangement

Example: How many ways can you rearrange the letters of my favorite word MATHEMATICS?

- There are 11 letters in total but not all are distinct so nPr is not applicable.
- Of the non-distinct letters, there are 2 M's; 2 A's; 2 T's.
- Of the 11 slots select 2 to place the M's.
- Of the 9 remaining slots, select 2 to place the A's.
- Of the 7 remaining slots, select 2 to place the T's.
- Now of the 5 remaining slots, select one at a time to place each of the remaining singletons (H, E, I, C, S)
- Apply the formula for nCr to see how to arrive at the answer and then generalize to determine how to compute the number of linear arrangements if not all objects are different.

$$= {}_{11}C_2 \cdot {}_9C_2 \cdot {}_7C_2 \cdot {}_5C_1 \cdot {}_4C_1 \cdot {}_3C_1 \cdot {}_2C_1 \cdot {}_1C_1$$

$$= \frac{11!}{2!\cancel{9!}} \cdot \frac{\cancel{9!}}{2!\cancel{7!}} \cdot \frac{\cancel{7!}}{2!\cancel{5!}} \cdot \frac{\cancel{5!}}{\cancel{4!}} \cdot \frac{\cancel{4!}}{\cancel{3!}} \cdot \frac{\cancel{3!}}{\cancel{2!}} \cdot \frac{\cancel{2!}}{\cancel{1!}} \cdot \frac{\cancel{1!}}{1!} = \frac{11!}{2!2!2!} = \frac{\text{total!}}{\text{repeats!}\,\text{rep!}\ldots}$$

$$= 4{,}989{,}600$$

#9: You design necklaces, earrings, and bracelets. You bring 4 identical necklaces, 10 identical earring pairs; and 5 identical bracelets to a fair to sell. In how many ways can you line up all of your jewelry on a display table?

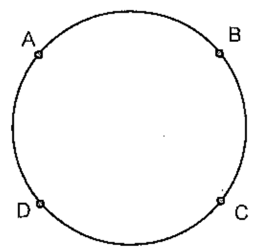
$4+10+5=19$ $\qquad \dfrac{19!}{4!\,10!\,5!} = 11{,}639{,}628$

#10: In how many ways can you rearrange the letters in the word "Cincinnati"?

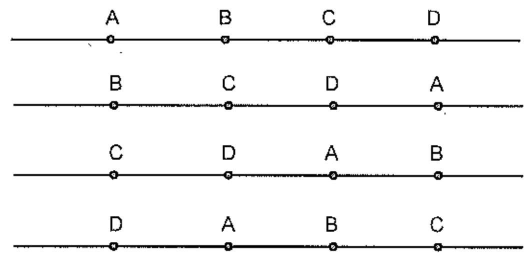
$$\frac{10!}{2!\,3!\,3!} = 50{,}400$$

Type #4: Circular Permutations:

In how many ways can you arrange 4 objects around the edge of a circular tray? The letters A, B, C, and D are arranged in a circle, as shown below.



Circular Permutations



Linear Permutations

Notice that all 4 distinct linear permutations of the letters A, B, C and D, give a single distinct circular permutation. Therefore, to find the number of circular permutations of 4 objects is:

Switch A&B A&D B&D
A&C B&C C&D $\Rightarrow$ 6 ways $3 \cdot 2 \cdot 1 = 3!$

In general, the number of ways to put n distinct objects in a circle is: $(n-1)!$

$\frac{n!}{n} = (n-1)!$

#11: At a Chinese restaurant seven different types of appetizers are served on a circular pu-pu platter. In how many ways can these seven appetizers be arranged?

$6! = 720$

Some to try:

1) 25 people are in a room. 10 are joggers and 15 are non-joggers. You are selecting four people to be in a survey. In how many ways can you select your four participants such that only one is a non-jogger?

$${}_{10}C_3 \cdot {}_{15}C_1 = 1{,}800$$

2) A caterer is arranging a row of desserts. The row will contain 8 platters of cookies, 5 trays of fruit, and 3 pies. In how many distinct ways can the cookies, fruit, and pies, be arranged in a row, if each type of dessert is of the same kind?

$8+5+3=16$ $$\frac{16!}{8!5!3!} = 720{,}720$$

3) At Casabellas there are ten different pizza toppings and two different sauce toppings (red or white). How many different types of pizzas can be made if you are going to chose 3 different pizza toppings and one sauce, either red or white?

$${}_{10}C_3 \cdot {}_{2}C_1 = 240$$

4) You have three different Algebra books and two different chemistry books. You need to arrange them in a line such that they alternate Algebra, Chemistry, Algebra, ..etc. How many ways can this be done?

$$\underset{A}{\underline{3}} \cdot \underset{C}{\underline{2}} \cdot \underset{A}{\underline{2}} \cdot \underset{C}{\underline{1}} \cdot \underset{A}{\underline{1}} = 12$$

5) How many ways can rearrange the letters of the word "MATH" in a circle?

no repeats $(4-1)! = 3! = 6$

6) How many ways can rearrange the letters of the word "MISSISSIPPI" in a line?

$$\frac{11!}{4!4!2!} = 34{,}650$$

7) A bag contains 10 different colored marbles. In how many ways can you select three?

$${}_{10}C_3 = 120$$

8) Your computer password has to have a letter, followed by three digits, followed by two letters. How many passwords are possible if the letters can repeat but the digits can not and the password must begin with a vowel $\underbrace{\text{(a-e-i-o-u)}}_{5}$?

$$\underset{L}{\underline{5}} \cdot \underset{D}{\underline{10}} \cdot \underset{D}{\underline{9}} \cdot \underset{D}{\underline{8}} \cdot \underset{L}{\underline{26}} \cdot \underset{L}{\underline{26}} = 2{,}433{,}600$$