Pre-Calculus CP 1 – Section 9.7 Notes **Name:** ____________________

Probability

Probability Models

- The sample space is the set of all possible outcomes for an experiment. Each element of a sample space has a probability between <u>0</u> (not going to happen!) and <u>1</u>(will absolutely happen!)
- The <u>sum</u> of **all** probabilities within a sample space must be equal to 1 (or 100%)

Using Addition with Probability "OR"

Example: Drawing a red card or a club are **mutually exclusive** because...

there is <u>no</u> overlap

If mutually exclusive then *P(A or B)* = P(A) + P(B)

Example: Drawing a card that is red or ace are **inclusive** events because....

there is 1 red ace → overlap

If inclusive then *P(A or B)* = P(A) + P(B) − P(A & B)

Using Multiplication with Probability "AND"

Definition: Two events are **independent** if the occurrence or non-occurrence of one event has no effect on the likelihood of the occurrence of the other event.

If *A* and *B* are independent events then *P(A and B)*= P(A) · P(B)

- Example: Mr. Kaplan and Mr. Bourque each get to select one student to send on a special trip to Washington D.C. If Hayden is in Bourque's class of 20 and Noah is in Kaplan's class of 24, what is the probability that they are **both** selected?

$$\frac{1}{20} \cdot \frac{1}{24} = \frac{1}{480}$$

Complements and Probability

The probability that an event does not occur, $\boxed{P(\bar{A}) = 1 - P(A)}$

Example: If I throw a die three times, what is the probability that I ~~never~~ at least one get a 5?

$P(\text{no 5 in 3 rolls}) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \frac{125}{216}$

$P(\text{at least 1 5}) = 1 - P(\text{no 5}) = 1 - \frac{125}{216} = \frac{91}{216}$

Using a Table with Probability

Example: For medical purposes, the managers of a company decide to record the blood type of all the employees. The results are shown in the table below.

	O	A	B	AB	totals
Women	8	5	4	2	19
Men	12	6	2	1	21
totals	20	11	6	3	40

a. Find the probability that a person has type B blood or type A blood.

$\frac{6+11}{40} = \frac{17}{40}$

b. Find the probability that a person is either a women or has type 0 blood.

$P(W) + P(O) - P(W \cap O) = \frac{19}{40} + \frac{20}{40} - \frac{8}{40} = \frac{31}{40}$

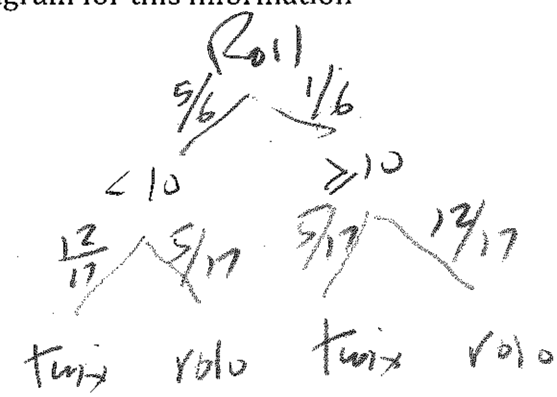
c. Find the probability that a person has type A blood **given** that a man was chosen.

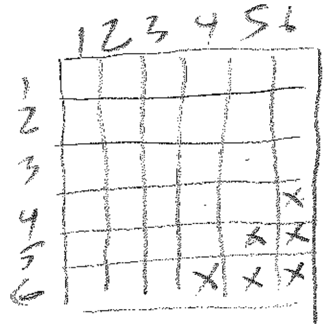
$\frac{6}{21} = \frac{2}{7}$

Using a Tree Diagram with Probability

You are playing a game where you roll two dice, and then use the number rolled to determine from which bag you get to pull a prize. If you roll a sum of less than 10, you get to pull from bag A, which has 12 twix bars and 5 rolos. Otherwise, you get to pull from bag B, which has 12 rolos and 5 twix bars.

a) Make a tree diagram for this information





b) Find the probability that you will get a twix bar.

$P(<10 \ \& \text{ twix})$ or $P(\geq 10 \ \& \text{ twix})$

$\frac{5}{6} \cdot \frac{12}{17} + \frac{1}{6} \cdot \frac{5}{17}$

$\frac{60}{102} + \frac{5}{102} = \frac{65}{102}$

c) Find the probability that you rolled a number less than 10 **given** that you got a twix bar:

$$\frac{P(<10 \ \& \text{ twix})}{P(\text{twix})} = \frac{\frac{5}{6} \cdot \frac{12}{17}}{\frac{65}{102}} = \frac{\frac{60}{102}}{\frac{65}{102}} = \frac{60}{65} = \frac{12}{13}$$

LOTS of EXAMPLES:

1. Mary Papadopoulos and her husband plan to have three children. What is the probability that the new Papadopoulos family will welcome three baby girls? Assume no twins or triplets.

$$\underset{1}{\frac{1}{2}} \cdot \underset{2}{\frac{1}{2}} \cdot \underset{3}{\frac{1}{2}} = \frac{1}{8}$$

2. What is the probability that the Papadopoulos family does not have three girls?

$$1 - \frac{1}{8} = \frac{7}{8}$$

3. A box contains 10 chocolate chip cookies and 8 sugar cookies. You are going to select 3 cookies from the box. What is the probability that you select exactly 2 chocolate chip cookies?

$$3\left(\underset{1}{\frac{10}{18}} \cdot \underset{2}{\frac{9}{17}} \cdot \underset{3}{\frac{8}{16}}\right) = 3\left(\frac{5}{9} \cdot \frac{9}{17} \cdot \frac{1}{2}\right) = 3\left(\frac{45}{306}\right) = 3\left(\frac{5}{34}\right) = \frac{15}{34}$$

total $= {}_{18}C_3$ exactly 2cc $= {}_{10}C_2 \cdot {}_8C_1$ $P = \frac{{}_{10}C_2 \cdot {}_8C_1}{{}_{18}C_3} = \frac{15}{34}$

4. A fair coin is tossed **four** times. Determine P(exactly one tail).

$$\frac{4}{2^4} = \frac{4}{16} = \frac{1}{4}$$

5. A fair coin is tossed **four** times. Determine P(no more than one tail)

$$P(1 \text{ tail or no tails}) = \frac{4}{16} + \frac{1}{16} = \frac{5}{16}$$

6. Suppose two fair dice are rolled and the two digits are added. What is the probability that the sum is an 11?

$$\frac{2}{36} = \boxed{\frac{1}{18}}$$

7. A fair die is rolled. What is the probability that the outcome is odd or a multiple of three?

$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$ $\quad P(\text{both}) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$P(\text{mult } 3) = \frac{2}{6} = \frac{1}{3}$

$P(\text{odd or mult of } 3) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \boxed{\frac{2}{3}}$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

8. You are going to roll a pair of dice three times. Find the probability that you get...

a) A sum of five on all three rolls.

2 & 3
3 & 2
4 & 1
1 & 4

$$\frac{4}{36} \cdot \frac{4}{36} \cdot \frac{4}{36} = \frac{1}{9^3} = \frac{1}{729}$$

b) A sum of five **at least** twice.

$P(\text{sum of 5 twice}) + P(\text{sum of 5 3 times})$

$$\frac{1}{9} \cdot \frac{1}{9} \cdot \frac{8}{9} + \frac{1}{9^3} = \frac{8}{729} + \frac{1}{729} = \frac{1}{81}$$

9. Consider the data set regarding the probability of owning a certain number of TV sets.

# of TV sets	0	1	2	3	4 or more
Probability	0.05	0.24	0.33	0.21	0.17

a) What is the probability that a randomly selected person owns less than 2 TV sets?

$P(<2) = .05 + .24 = .29$

b) What is the probability that a randomly selected person owns one or more TVs?

$P(\geq 1) = 1 - P(<1) = 1 - .05 = .95$

10. A bag of M&Ms contains 15 reds, 24 browns, 11 yellows, and 10 blues. You are going to reach in to the bag and select one M&M at a time without replacement.

a) What is the probability that you select either a red or blue?

$\frac{15}{60} + \frac{10}{60} = \frac{25}{60} = \frac{5}{12}$

b) What is the probability that you select a brown, then a yellow, then a blue, then another yellow?

$\frac{24}{60} \cdot \frac{11}{59} \cdot \frac{10}{58} \cdot \frac{10}{57} = \frac{26,400}{11,703,240} = \frac{220}{97,527}$

c) What is the probability that you select a red, then a blue or yellow, then a yellow?

red, b, y or red, y, y

$\frac{15}{60} \cdot \frac{10}{59} \cdot \frac{11}{58} + \frac{15}{60} \cdot \frac{11}{59} \cdot \frac{10}{58}$

$\frac{55}{6844} + \frac{55}{6844} = \frac{55}{3422}$